Ⅲ - B 28 Critical loads of pile foundation

Faculty of Eng. Yamaguchi Univ. Fusanori MIURA*
Doctor Course Yamaguchi Univ. Zhu Yuanyuan**

I. Introduction

According to the recent post-earthquake investigation reports, we can see plenty of damage pattern of piles and pile foundations. Some of them were caused by strong ground motions and some of them liquefaction-induced large ground displacements. This recognition has important meaning for the design of the structures and requires that the stability analysis models of pile foundation should be devised to evaluate these potential problems. In this paper, we obtained critical loads for a linear problem of single pile foundation with initial deflection as a first step. The results show that when initial deflection is increased, the critical load of pile foundation is correspondingly decreased.

II. Governing equation

Assuming that the pile foundation has initial deformation of $u_0(s)$ in x direction and $w_0(s)$ in y direction, $\Gamma_0:\{(x,y)|x = s+u_0(s),\ y = w_0(s),\ 0 \le s \le l\}$ is the domain occupied by the pile before additional deformation caused by external forces, in which $s \in [0,l]$ is the coordinate along the pile axis and l is the length of the pile (see Fig. 1).

$\theta_0(s)$ is an angle between tangent line and x-axis at the point C. It is not difficult to get

$$\cos\theta_0 = 1 + u_0', \quad \sin\theta_0 = w_0' \tag{1}$$

where, $u_0' = \dfrac{du_0}{ds}$, $w_0' = \dfrac{dw_0}{ds}$.

Let $q(s)$ be the reaction of soil, and the pile is subjected to $[-P,0]$ at the point of $s = l$. After applying the force, an arbitrary point $C(s+u_0, w_0)$ in the Γ_0 moves to point $C'(s+u_0+u, w_0+w)$, and the pile occupies the domain of $\Gamma:\{(x,y)|x = s+u_0(s)+u(s),\ y = w_0(s)+w(s),\ 0 \le s \le l\}$ (see Fig. 2).

$\theta(s)$ is an angle between tangent line and x-axis, and we have

$$\cos\theta = 1 + u_0' + u' = \cos\theta_0 + u', \quad \sin\theta = w_0' + w' = \sin\theta_0 + w' \tag{2}$$

in which, $u(s)$ and $w(s)$ are displacements of x-direction and y-direction respectively. $N(s), Q(s), M(s)$ are the axial force, shear force, and bending moment at point C', respectively.

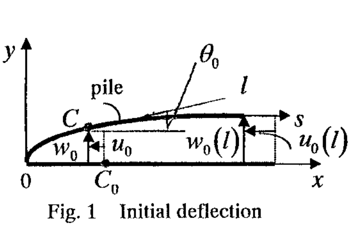


Fig. 1 Initial deflection and coordinate system

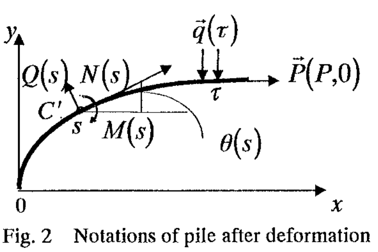


Fig. 2 Notations of pile after deformation caused by external forces

The reaction vector $\vec{q}(s)$ is obtained by multiplying the soil spring coefficient k_h and deformation of the pile at point $C'(s+u_0+u, w_0+w)$ as follows;

Key Words: pile foundation, stability analysis, critical load
* Yamaguchi University, Tokiwadai, Ube, 755-8611, Japan, Tel (Fax): (0836)-35-9483
** Yamaguchi University, Tokiwadai, Ube, 755-8611, Japan

$$\vec{q}(\tau) = k_h\left(u(\tau)\sin\theta_0 - w(\tau)\cos\theta_0\right)\begin{pmatrix} -\sin\theta_0 \\ \cos\theta_0 \end{pmatrix} \tag{3}$$

In this equation, we assumed that the direction of vector $\vec{q}(s)$ is normal to the pile at the initial state.

From the equilibrium and constitutive equation of the pile, it is not difficult to get

$$\begin{aligned} &EI(\theta''' - \theta_0''') + P\cos\theta\theta' - k_h\left(u(s)\sin\theta_0(s) - w(s)\cos\theta_0(s)\right)\cos(\theta_0(s) - \theta(s)) \\ &- \left\{k_h\int_s^l \left(w(\tau)\cos\theta_0(\tau) - u(\tau)\sin\theta_0(\tau)\right)\sin(\theta_0(\tau) - \theta(s))d\tau\right\}\theta'(s) = 0 \end{aligned} \tag{4}$$

in which, EI is the flexural rigidity of the pile.

We assume both side of the pile are simply-supported (S.S), which is

$$\begin{cases} u(0) = w(0) = 0, \ \theta'(0) = \theta_0'(0) \\ w(l) = 0, \ \theta'(l) = \theta_0'(l) \end{cases} \tag{5}$$

The nonlinear boundary value problem are (2),(4),(5).

From the differentiation of (2) by s, we obtain $\cos\theta\theta' = w'' + w_0''$. When the problem tends small deformation, that is $w(s), u(s), \theta(s) - \theta_0(s)$ are small, we have dimensionless governing equation as follows;

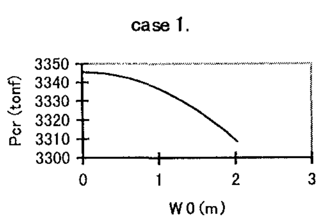
$$\begin{aligned} &W^{(4)} + \lambda W'' + \alpha\cos\theta_0 W = -\lambda W_0'' \\ &W(0) = W''(0) = W(l) = W''(l) = 0 \end{aligned} \tag{6}$$

where, $t = \frac{s}{l}$, $W = \frac{w}{l}$, $U = \frac{u}{l}$, $\lambda = \frac{Pl^2}{EI}$, $\alpha = \frac{k_h l^4}{EI}$ are the dimensionless factors.

The problems (6) are valid under the condition of which the initial deflection may be large, but the deformation is small ($w(s), u(s), \theta(s) - \theta_0(s)$ are small).

III. Critical load of pile

By using numerical calculating of problem (6), we can get the critical load of pile as shown in Fig. 3, in which $w_0(0) = 0, \theta_0(s) = const$. Parameters used are summarized in Table 1. From these figures, we can see when initial deflection is increased, the critical load of pile foundation is decreased correspondingly.



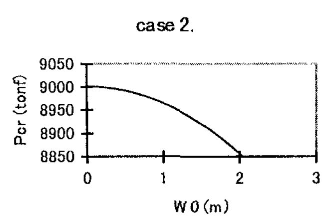


Fig. 3 Critical load of pile foundation

Table 1. Model parameters

case	Diameter(mm)	EI(tonf-m^2)	k_h (tonf/m^2)	l (m)
1	400	3983	700	10
2	600	19337	700	10