Structural Eng./Earthquake Eng., JSCE, Vol. 16, No. 1, 1s—10s, 1999 January
(J. Struct. Mech. Earthquake Eng., JSCE, No. 612/ I —46)

DYNAMIC RESPONSE OF STRUCTURES SUPPORTED ON ELLIPTICAL ROLLING RODS

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In this paper, effectiveness of a new base isolation system i.e. elliptical rolling rods between the base and the foundation of structure is investigated. Equations governing the motion of a multi-storey shear type building supported on the elliptical rolling rods are derived. The dynamic response of the system to both harmonic and real earthquake ground motions is obtained by integrating the incremental equations along with an iterative technique. The iterations are required due to non-linear force-deformation behaviour of the elliptical rolling rods. Effectiveness of the elliptical rolling rods is studied by comparing the response between isolated and corresponding fixed base system. In addition, a parametric study is conducted to investigate the effects of important parameters on the effectiveness of elliptical rolling rods. The parameters considered are: the fundamental time period of the superstructure, the coefficient of rolling friction and the eccentricity of the elliptical rolling rods and the frequency content of ground motion. It is shown that the elliptical rolling rods are quite effective in reducing the dynamic response of the system without undergoing the large displacements or living the large residual base displacement.

***Key Words*: *base isolation, elliptical rolling rods, eccentricity, earthquake, friction*.**

1. INTRODUCTION

Base isolation is an aseismic design approach in which an isolation system is used to decouple a building from the ground so that the damaging horizontal component of earthquake ground motion cannot be transmitted into the building. The main concept in base isolation is to reduce the fundamental frequency of structural vibration to a value lower than the predominant energy containing frequencies of earthquake ground motions. The other purpose of an isolation system is to provide a means of energy dissipation and thereby, reducing the transmitted acceleration into the superstructure. Accordingly, by using base isolation devices in the foundations, the structure is essentially uncoupled from the ground motion during earthquakes. The effectiveness of various types of base isolators in limiting the earthquake forces has been widely studied[1)2)3)4)5)]. Further, Buckle and Mayes[6)] and Jangid and Datta[7)] provided an excellent preview of earlier and recent development on base isolation systems.

In the past, several base isolation devices have been developed and some of them are now practically provided in several building in USA, Japan, New Zealand and other Countries. Some of the important devices are: the laminated rubber bearings with and without lead core, alexisismon isolation system[8)], "Electricite de France" (EDF) system[9)], earthquake barrier system[10)], the resilient-friction base isolator (R-FBI) system[1)], the sliding resilient-friction (SR-F) system[2)] and the friction pendulum system[11)]. Among these devices, the frictional base isolators are more popular. The most attractive feature of this type of isolator is that the frictional force is natural and powerful energy dissipation device and effective for a wide frequency range of input ground motion. Recently, a new system of a free circular rolling rods between the base and foundation of structure in two orthogonal directions was proposed by Lin and Hone[12)]. The main advantage of rolling rod is its low value of rolling friction coefficient, as a result, a very low earthquake forces are transmitted to superstructure. However, such a system suffer from the fail-safe device resulting in large peak and residual base displacements. To overcome this, Lin et al.[13] proposed the cantilever beam along with rolling rods to provide the restoring force in the isolation system. However, such a fail-safe device requires an additional cost for base isolation.

The other alternative to make the rolling rods fail-safe is by using the elliptical shape instead of the circular one. Due to eccentricity of the elliptical rolling rods a restoring force is developed which brings back the structure to its original position. In the present study, response of a multi-storey shear type structure supported on elliptical rolling rods to harmonic and real earthquake ground motion is investigated with the specific objectives as: (1) to present a theoretical formulation for dynamic analysis of a base-excited structure supported on the elliptical rolling rods; (2) to study the effectiveness of elliptical rolling rods as a base isolation system; (3) to compare the response of the structure supported on elliptical rolling rods to that with circular rolling rods; and (4) to study the effects of important parameters on the effectiveness of the elliptical rolling rods. The various important parameters considered are the ratio of harmonic excitation frequency to the fundamental frequency of superstructure, the fundamental time period of the superstructure and the coefficient of rolling friction and the eccentricity of the elliptical rolling rods.

2. STRUCTURAL MODEL

Fig. 1 shows the structural model of an idealised N-storey shear type structure mounted on the elliptical rolling rods in two orthogonal directions. The elliptical rolling rods are provided between the base mass and the foundation of the structure. The low value of coefficient of rolling friction and the eccentricity of the elliptical rolling rods provide the desired isolation effects in the structure. The low coefficient of friction ensures the transmissibility of a limited earthquake force into the superstructure. On the other hand, the eccentricity of the elliptical rolling rods provides a restoring force, which reduces the peak base displacement as well as brings back the structure to its original position after the earthquake. Although, the elliptical rolling rods may not be a practical system for large structure but ellipsoidal bearings can be used for the isolation of structures in two horizontal directions. The various assumptions made for the structural model under consideration are: (1) the coefficient of rolling friction between the rolling rods and the base mass

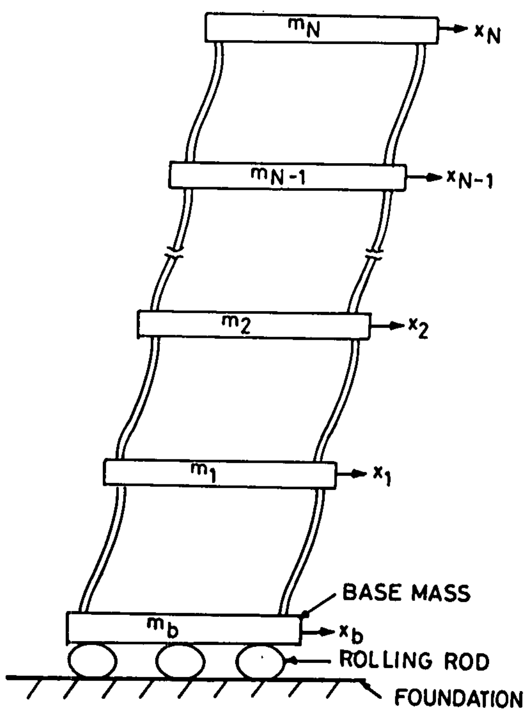

Fig. 1 Structural model of base-isolated structure.

remains constant throughout the motion of the structure (i.e. the friction coefficient is independent of velocity, pressure and the instantaneous radius of the elliptical rods); (2) the superstructure remains elastic during earthquake excitation. This is a reasonable assumption, since the purpose of elliptical rolling rods is to reduce the inertia forces in such a way that the superstructure remains within the elastic limits; (3) no overturning or tilting will occur in the superstructure due to rolling over the elliptical rolling rods; and (4) the effects of vertical component of the ground acceleration are neglected. At each floor and base mass one lateral dynamic degree-of-freedom is considered. Therefore, for the N-storey base-isolated building the dynamic degrees-of-freedom are $N+1$.

During the ground excitation, the elliptical rolling rods undergo translational (in horizontal and vertical directions) and the rotational displacement. Therefore, they have three degrees-of-freedoms as shown in Fig. 2(a). Note that the displacement and rotation of the elliptical rolling rods are dependent on the displacement of the base mass. Referring to Fig. 2(b), the translational and rotational displacements of the elliptical rolling rods are given by

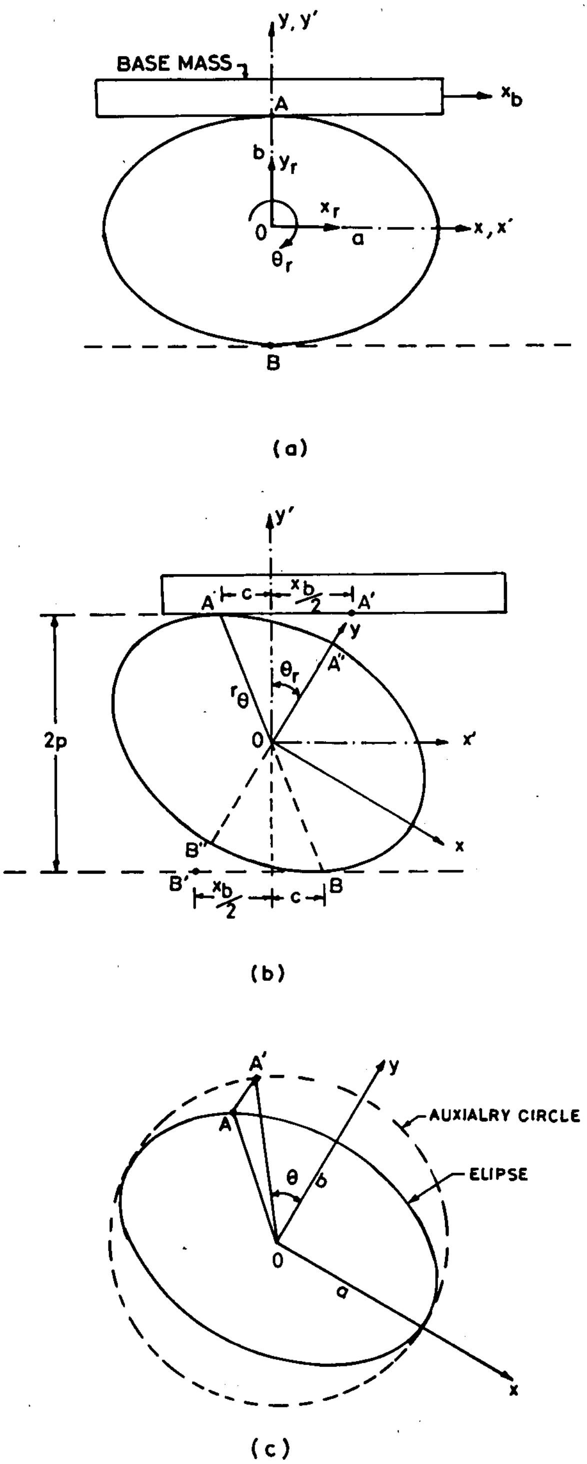

Fig. 2 Relation between the displacements of the base mass and elliptical rolling rods.

$$x_r = \frac{x_b}{2} \tag{1}$$

$$y_r = p - b \tag{2}$$

$$\theta_r = tan^{-1}(\frac{b}{a} tan\,\theta) \tag{3}$$

where x_r and y_r are the horizontal and vertical displacement of the elliptical rolling rods, respectively (relative to the ground); θ_r is the rotation of the elliptical rolling rods; x_b is the horizontal displacement of the base mass; a and b are the larger and smaller radius of the elliptical rolling rods as shown in Fig. 2(a); $p = a\,sin\,\theta\,sin\,\theta_r + b\,cos\,\theta\,cos\,\theta_r$ is the half of the vertical distance between lower and upper contact points (refer points A and B in the Fig. 2(b)) of the rolling rods; θ is the eccentric angle such that the co-ordinates of point A are ($-a\sin\theta$, $b\cos\theta$) and hence the distance $OA = \sqrt{a^2\,sin^2\,\theta + b^2\,cos^2\,\theta}$ as shown in Fig. 2(c); and e is the eccentricity of the elliptical rods given by

$$e = \sqrt{\frac{a^2 - b^2}{a^2}} \tag{4}$$

By equating the distances AA' and AA'' in the Fig. 2(b) a relation between the angle θ and the base displacement x_b is expressed as

$$\frac{x_b}{2} + c = \int_0^{\theta} a\sqrt{1 - e^2\,sin^2\,\phi}\,d\phi \tag{5}$$

where $c = a\,sin\,\theta\,cos\,\theta_r - b\,cos\,\theta\,sin\,\theta_r$ is the half of the horizontal distance between the upper and lower contact points.

Note that the right hand side of Eq. (5) contains special function which is known as elliptic integral. For a given value of e and θ, the value of the integral are available in standard charts and tables[14]. In the present study, for a particular value of the base displacement, x_b, the value of θ is obtained by trial and error along with the use of standard tables.

(1) Governing equations of motion

Equations of motion of the N-storey linear shear type superstructure subjected to base excitation is written in the matrix form by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{1\})(\ddot{x}_b + \ddot{x}_g) \tag{6}$$

where $[M]$, $[K]$ and $[C]$ are the mass, stiffness and damping matrices of the superstructure, respectively of the order $N \times N$; $\{x\} = \{x_1, x_2, ..., x_N\}^T$ is the displacement vector of the superstructure; x_j $(j=1,2,...,N)$ is the lateral displacement of the jth floor relative to the base mass; $\{1\} = \{1, 1, 1, ..., 1\}^T$ is the influence coefficient vector; and $\ddot{x}_g$ is the ground acceleration.

In Fig. 3(a) the free body diagram of the base mass is shown and by considering the equilibrium of various forces the governing equation of motion for base mass is given by

$$m_b\ddot{x}_b + F_s + F_b - c_1\dot{x}_1 - k_1x_1 = -m_b\ddot{x}_b \qquad (7)$$

where m_b is the mass of the base raft; c_1 and k_1 are the damping and stiffness of the first-storey, respectively; F_s is the frictional force between the rolling rods and the base mass; and F_b is the force transmitted to the base mass due to inertia forces of the rolling rods.

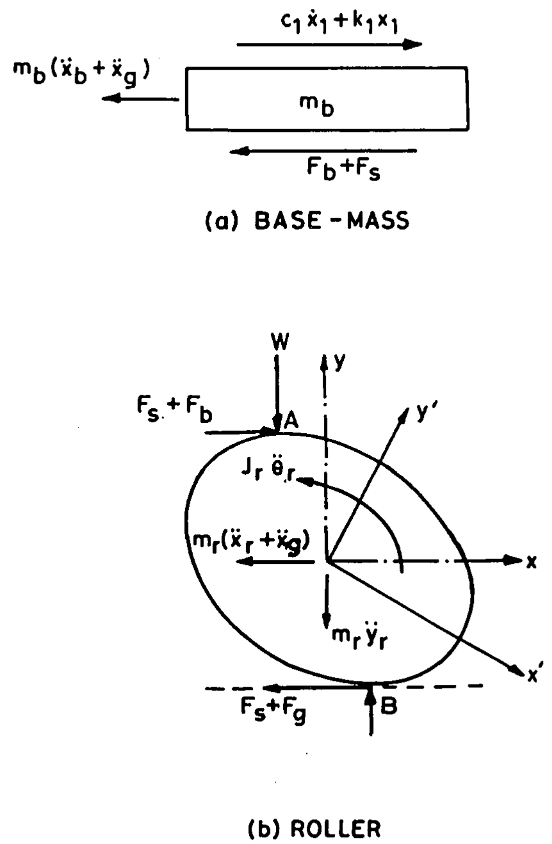

Fig. 3 Free body diagrams of the base mass and elliptical rolling rods.

The inertia forces of the elliptical rolling rods are balanced by the forces F_b and F_g (where F_g is the force between the rolling rods and foundation). The total horizontal force between the elliptical rolling rods and the base mass is F_b+F_s. The magnitude of the force F_b can be evaluated by considering the equilibrium of various inertia forces acting on the rolling rods (refer Fig. 3(b)). Thus, the forces F_s and F_b are expressed as

$$F_s = \left(\sum_j m_j + m_b\right)\left(g + \ddot{y}_b\right)\left(\mu sgn(\dot{x}_b) + \frac{c}{p}\right)$$
$$(-90° \leq \theta \leq 90°) \qquad (8)$$

$$F_b = \frac{1}{2p}J_r\ddot{\theta}_r + m_r\ddot{y}_r\frac{c}{p} + \frac{1}{2}m_r\left(\ddot{x}_r + \ddot{x}_g\right)$$
$$(-90° \leq \theta \leq 90°) \qquad (9)$$

where m_r and J_r are the sum of masses and the moment of inertia of all elliptical rods; m_j $(j=1,2,...,N)$ is the mass of the jth floor; μ is the coefficient of rolling friction between the base mass and the elliptical rolling rods; sgn denotes the signum function i.e. $sgn(\dot{x}_b) = \dot{x}_b/|\dot{x}_b|$; g is the acceleration due to gravity; and $\ddot{y}_b$ is the vertical acceleration of the base mass due to the vertical movement of the elliptical rolling rods, which is twice the vertical acceleration at the centre of the rods, $\ddot{y}_r$.

It is to be noted that in the Eq. (8) the first quantity inside the right bracket is due to the rolling friction and the second quantity (i.e. c/p) in due to restoring force provided by the eccentricity of the elliptical rolling rods. Further, for the case of circular rolling rods (i.e. $e = 0$ and for which $c/p = 0$) this expression reduces to the corresponding expression given by Lin and Hone[12].

(2) Conditions for rolling and non-rolling states

As long as the velocity and acceleration of the base mass are zero (i.e. $\dot{x}_b = \ddot{x}_b = 0$) the system does not roll over the rolling rods. In this phase of motion, the frictional force between the elliptical rolling rods and the base mass is greater than the total inertial force generated in the structure. The system starts rolling as the horizontal force between the base mass and the rolling rods exceeds the limiting value. Thus, the non-rolling state exists if

$$F_s > \left| F_b + m_b(\ddot{x}_b + \ddot{x}_g) - c_1\dot{x}_1 - k_1x_1 \right| \quad (10)$$

and

$$\mu sgn(\dot{x}_b) + \frac{c}{p} < \mu_s \quad (11)$$

where μ_s is the coefficient of sliding friction between the base mass and the rolling rods. The condition expressed in the Eq. (11) ensures that the sliding of structure over the rolling rods would not occur.

Failure of the non-rolling condition given by Eq. (10) indicates the occurrence of sliding phase and the Eq. (7) is to be considered for obtaining the dynamic response of the system. During the rolling phase of system whenever the relative velocity of the base mass becomes zero (i.e. $\dot{x}_b = 0$), the condition for non-rolling phase must be checked in order to determine whether the rolling rods are in the rolling phase or stick to the foundation.

(3) Incremental solution procedure

The governing equations of motion of the structure are non-linear (refer Eqs. (6) to (9)). As a result, the equations of motion are to be solved in the incremental form. For the present study, the solution of equations of motion is obtained by Newmark's method assuming linear variation of acceleration over the short time interval, δt. The solution of incremental equations requires the determination of the incremental forces (δF_s and δF_b) in each time step. It is to be noted that the δF_s and δF_b involve the response of the system at time $t+\delta t$. Therefore, an iterative procedure is employed to obtain the incremental force in each time step. The response of the sliding structures is quite sensitive to the time interval, δt and initial conditions at the beginning of rolling and non-rolling phases. Therefore, for the time intervals during which transition from one phase to another phase occurred, the time interval was reduced to minimise the unbalanced forces. In this study, the results are obtained with time interval $\delta t = 0.001$ sec for both phases and $\delta t = 0.001/100$ in the neighbourhood of the transition of the phase. The number of iterations in each time step are taken as 10 to determine the incremental frictional forces at the sliding support. In order to ensure that the structure rolls over the rolling rods without sliding the sliding coefficient of friction, μ_s is taken as 10 times the rolling friction coefficient.

3. NUMERICAL STUDY

Dynamic response of a flexible superstructure supported on the elliptical rolling rods for harmonic and real earthquake (S69E component of Taft 1952 earthquake) motion is investigated. The response quantities of interest are the top floor absolute acceleration of the superstructure ($\ddot{x}_N + \ddot{x}_b + \ddot{x}_g$) and the relative displacement of the base mass (x_b). The former is directly proportional to the forces (shear force and bending moments) that are exerted in the superstructure due to ground motion. The latter is a measure of displacement between the isolated structure and the ground that is crucial for the design of the elliptical rolling rods system. The harmonic ground excitation is assumed to be as $\ddot{x}_g = a_0 sin(\Omega t)$ (where a_0 = amplitude of the harmonic acceleration taken as $0.5g$; g = acceleration due to gravity; and Ω = harmonic excitation frequency). Note that the response of a non-linear system to different harmonic frequencies gives considerable insight into the dynamic characteristics of the system, which may be helpful in interpreting the response to the other type of excitation.

For the present study, the mass matrix of the superstructure, $[M]$, is diagonal and characterised by the mass of each floor which is kept constant. Also, for simplicity the stiffness of each floor, k is kept constant. The parameter k is selected to give the required fundamental time period of fixed base structure ($T_s = 2\pi/\omega_s$; ω_s is the fundamental frequency of fixed base structure). The damping matrix of the superstructure, $[C]$, is not known explicitly. It is constructed by assuming the modal damping (ξ_s) which is kept constant in all modes of vibration. The important parameters which may significantly influence the response of the system are: the ratio of harmonic excitation frequency to the fundamental frequency of fixed base structure (Ω/ω_s); the fundamental time period of fixed base structure (T_s); the coefficient of rolling friction of rolling rods (μ); and the eccentricity of the elliptical rolling rods (e). The other parameters of interest may be the ratio of the base mass and superstructure floor mass (m_b/m); and the ratio of the mass of the rollers to the superstructure floor mass (m_r/m). However, it is shown by Londhe[15] that the response of the system is not significantly influenced by the mass ratios m_b/m and m_r/m, as a result, in the present study these parameters are held constant. The values of the parameters held constant are: ξ_s =

2 %, $N = 5$, $m_b/m = 1$, $m_r/m = 0.05$, $m = 1000$ kg and $a = 200$ mm.

(1) Response to harmonic excitation

Fig. 4 shows the time variation of the top floor absolute acceleration and the base displacement under harmonic base excitation. The parameters considered are: Ω/ω_S=0.796, T_S = 0.5 sec, e = 0 and 0.5 and μ = 0.01. The peak value of the top floor absolute acceleration is reduced to 2.15 m/sec^2 for elliptical rods and 1.94 m/sec^2 for circular rods from 20.1 m/sec^2 for the corresponding fixed base system indicating significant reduction in the absolute acceleration. This shows that the rolling rods are quite effective in reducing the response of the superstructure. Further, the peak base displacement is significantly low for the case of the elliptical rods as compared to the circular rods and also the residual displacement is zero in case of the elliptical rods. Thus, by using the elliptical rolling rods, the design base displacement can be reduced significantly as compared to the circular rolling rods.

In order to study the effects of the eccentricity of the elliptical rolling rods, the variation of peak top floor absolute acceleration and the base displacement to different eccentricities (i.e. e = 0, 0.25, 0.5 and 0.75) is plotted in Fig. 5 for T_S = 0.5 sec and μ = 0.01. It is seen from the figure that the peak top floor absolute acceleration is not significantly influenced by the eccentricity of the rolling rods. Further, the peak absolute acceleration also remains insensitive to the frequency content of ground excitation. Thus, the rolling rods can be effectively used for different kinds of soil conditions. On the other hand, the peak base displacement increases with the decrease of the eccentricity indicating that the elliptical rolling rods are more effective for base isolation as compared to the circular (e = 0) rolling rods.

In Fig. 6, the variation of peak top floor absolute acceleration and the base displacement is shown against different values of coefficient of rolling friction of the rods for T_s = 0.5 sec and e = 0.5. As the friction coefficient increases the absolute acceleration of the superstructure increases rendering to less effectiveness of the rolling rods. Thus, the effectiveness of rolling rods is reduced for higher value of coefficient of friction. On the other

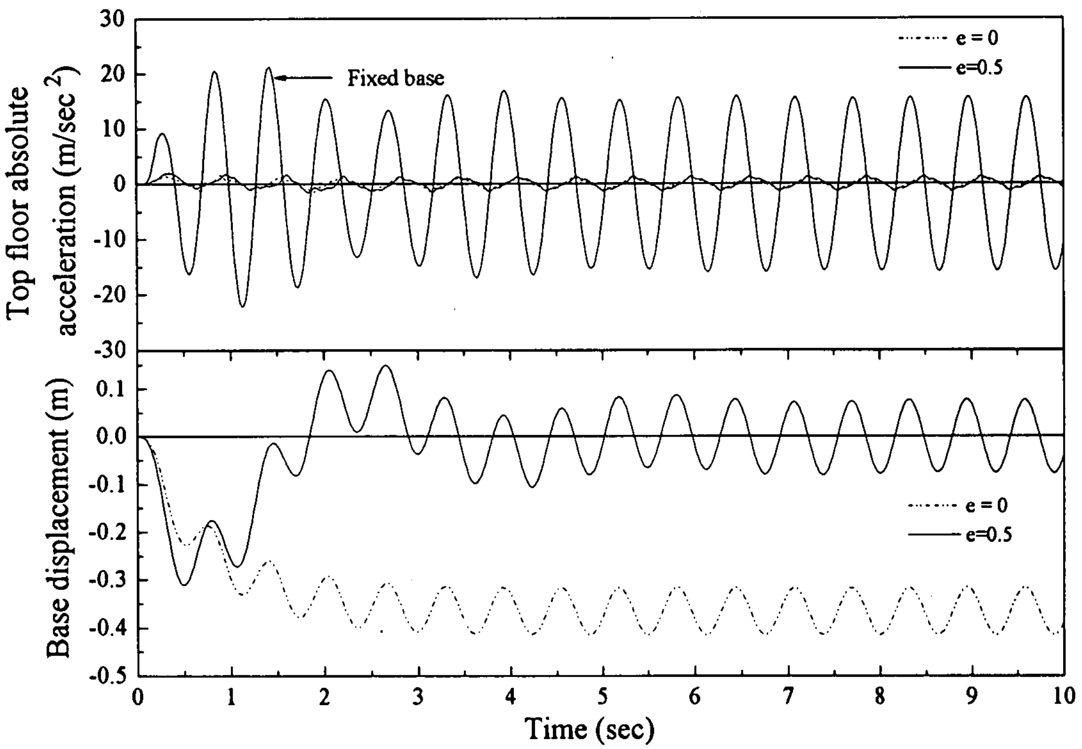

Fig. 4 Time variation of the top floor absolute acceleration and base displacement to harmonic support motion (Ω/ω_s = 0.796, T_s = 0.5 sec, e = 0, 0.5 and μ = 0.01).

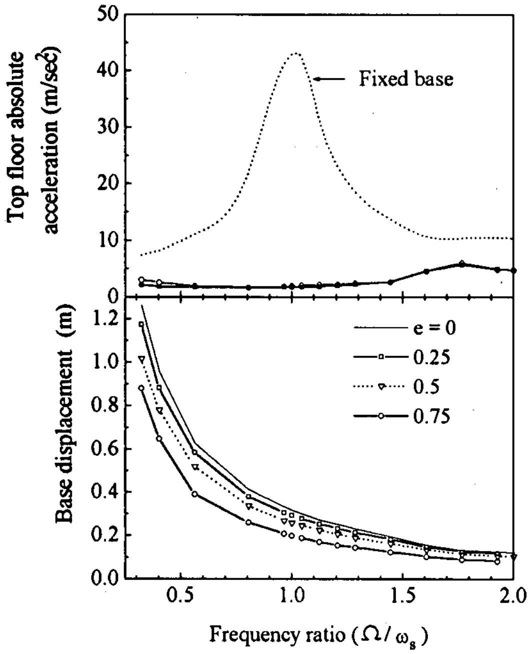

Fig. 5 Effects of eccentricity of the elliptical rolling rods on the variation of the peak top floor absolute acceleration and base displacement to harmonic base excitation (T_s = 0.5 sec and μ = 0.01).

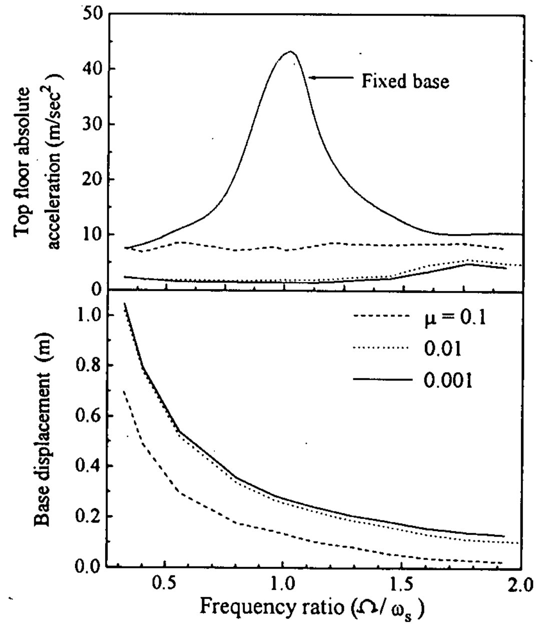

Fig. 6 Effects of coefficient of friction of the elliptical rolling rods on the variation of the peak top floor absolute acceleration and base displacement to harmonic base excitation (T_s = 0.5 sec and e = 0.5).

hand, the base displacement is significantly reduced for higher value of coefficient of friction. Note that the absolute acceleration transmitted to the superstructure can be reduced at the expense of increasing relative displacement of rolling rods. However, the base displacement has a practical limitation. Therefore, in designing the rolling rods system a compromise is made between transmitted absolute acceleration and the relative base displacement at the foundation level.

(2) Response to Earthquake Excitation

Time history of the top floor absolute acceleration and base displacement of structure with fixed base and supported on rolling rods to Taft, 1952 earthquake ground motion is shown in Fig. 7 for T_s=0.5 sec, e=0, and 0.5 and μ=0.01. The absolute acceleration in case of the system with rolling rods is significantly reduced as compared to that of the fixed base system. The peak value of the top floor absolute acceleration is reduced to 1.5 m/sec^2 for the elliptical rods (e = 0.5) and 1.26 m/sec^2 for circular rods (e = 0) from 4.79 m/sec^2 for the corresponding fixed base system. Further, the peak base displacement is significantly low for the case of the elliptical rods as compared to the circular rods. Moreover, there is no residual base displacement in case of the elliptical rolling rods. Thus, by using the elliptical rolling rods, the design base displacement can be reduced significantly.

The eccentricity of the elliptical rolling rods induces vertical acceleration into the superstructure (refer Fig. 2) which can be crucial for the members from the stability point of view. For finding out the magnitude of vertical acceleration induced in the superstructure ($\ddot{y}_b$), its time variation under Taft earthquake excitation is shown in Fig. 8 for T_s = 0.5 sec, e = 0 and 0.5 and μ = 0.01. The peak vertical acceleration induced is 0.0105g which is quite low as compared to acceleration due to gravity. Thus, the vertical acceleration induced in the superstructure due to eccentricity of elliptical rolling rods may not be quite significant. In the Fig. 8 the time variation of the parameters θ, F_b and F_s is also plotted. As expected the variation of θ is similar to that of x_b in Fig. 7. Further, the plot of F_b and F_s indicates that the condition for Eqs. (8) and (9) is satisfactory.

The effects of eccentricity and the friction coefficient of the rolling rods on the top floor absolute acceleration and the base displacement are shown in Figs. 9 and 10, respectively. The effects of

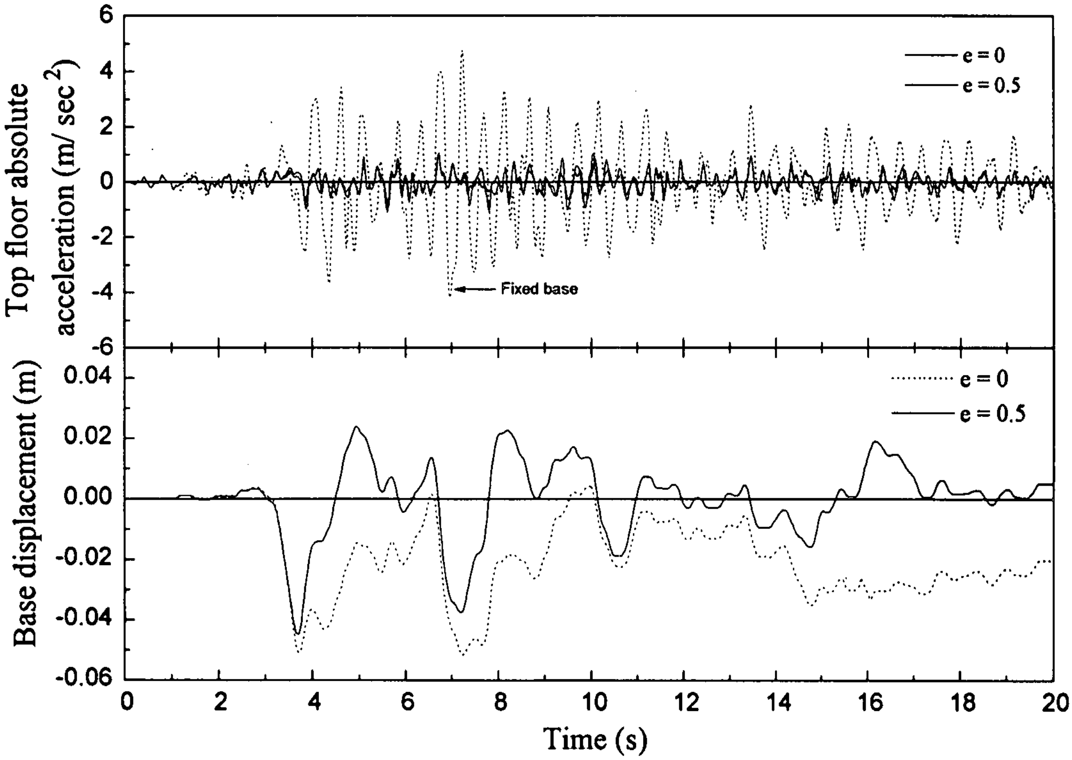

Fig. 7 Time variation of the top floor absolute acceleration and base displacement to Taft 1952 earthquake ground motion (T_s = 0.5 sec, e = 0.5 and μ = 0.01).

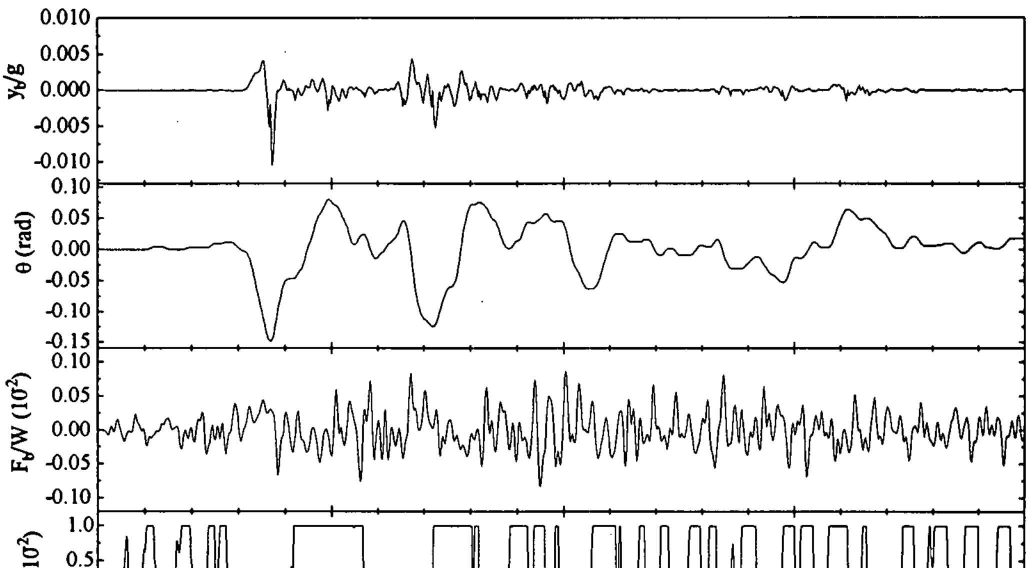

Fig. 8 Time variation of $\ddot{y}_b$, θ, F_b and F_s to Taft 1952 earthquake ground motion (T_s = 0.5 sec, e = 0.5 and μ = 0.01).

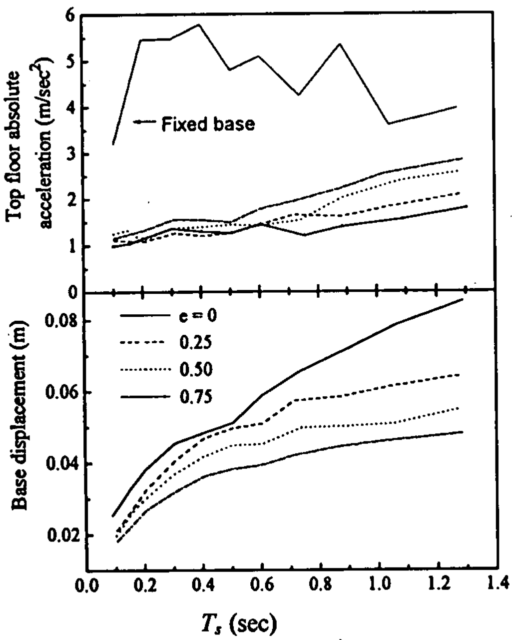

Fig. 9 Effects of eccentricity of the elliptical rolling rod on the variation of the peak top floor absolute acceleration and base displacement to Taft 1952 earthquake ground motion (μ = 0.01).

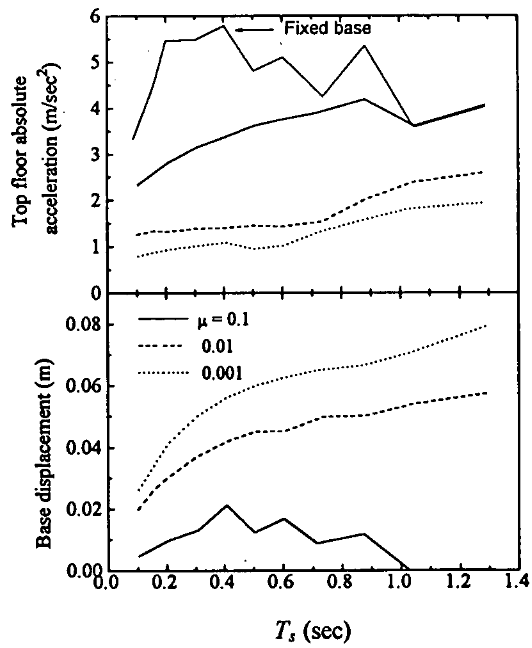

Fig. 10 Effects of coefficient of friction of the elliptical rolling rod on the variation of the peak top floor absolute acceleration and base displacement to Taft 1952 earthquake ground motion (e = 0.5).

these parameters on the effectiveness of rolling rods are the same as that observed for harmonic excitation. However, the top floor absolute acceleration and base displacement increases as the fundamental period of the superstructure increases indicating the less effectiveness of rolling rods systems. Thus, the effectiveness of the elliptical rolling rods decreases with the increase of the flexibility of the superstructure.

It is to be noted that the elliptical rolling rods may show worse response for the case when $|\theta|>90°$ resulting in the significant permanent base displacement. This situation may arise during strong earthquake shaking or predominant low frequency earthquake ground motion.

4. CONCLUSIONS

The dynamic response of a multi-storey shear type building supported on the elliptical rolling rods is studied to harmonic and real earthquake ground motion. The response of the system is analysed to investigate the effectiveness of the elliptical rolling rods as a base isolation system. The desired isolation effects are achieved due to the eccentricity and low rolling friction coefficient of the elliptical rolling rods. A parametric study is conducted to study the effects of important parameters on the effectiveness of the elliptical rods. From the trends of the results of the present study, the following conclusion may be drawn:

1) The structure can be well isolated with the help of elliptical rolling rods between the base and the foundation of the structure from the ground motions. The response of the structure is found to be insensitive to the frequency content of the ground motion and therefore, the rolling rods can be effectively used for all kinds of soil conditions.
2) The elliptical rolling rods are found to be better than circular rolling rods. The elliptical rolling rods provide significant reduction in the peak and residual base displacement. However, in some cases the elliptical rolling rods may show worse response than the circular rods.
3) The effectiveness of elliptical rolling rods increases with the decreasing values of the rolling coefficient of friction. However, the base displacement increases with the decreasing values of the coefficient of friction.
4) The peak base displacement decreases with the increasing values of the eccentricity of the elliptical rods. However, the eccentricity of the

elliptical rolling rods does not have much influence on the absolute acceleration of the superstructure.

5) The effectiveness of the elliptical rolling rods decreases with the increase of the flexibility of the superstructure.

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(Received March 18, 1997)