(第25回地震工学研究発表会講演論文集（1999年７月))

# STATISTICAL STUDY OF SPATIAL VARIATION OF RESPONSE SPECTRUM

Hideji KAWAKAMI[1], Shailendra SHARMA[2] and Hidenori MOGI[3]

[1] Member of JSCE, Dr. Eng., Professor, Dept. of Civil Eng., Saitama University (Shimo-ohkubo, Urawa, 338- 8570, Japan)
[2] M. Eng., Dept. of Civil Eng., Saitama University (Shimo-ohkubo, Urawa, 338- 8570, Japan)
[3] Member of JSCE, Dr. Eng., Research Associate, Dept. of Civil Eng., Saitama University (Shimo-ohkubo, Urawa, 338- 8570, Japan)

Spatial variation of acceleration response spectra is examined using strong motion records for a large number of events from dense accelerometer arrays at Chiba in Japan and SMART-1 in Lotung, Taiwan. The effects of earthquake component, structural damping, earthquake magnitude, focal depth, epicentral distance, structural time period, and station separation on the intra- event variation of response spectra are examined first through an empirical analysis and then through a least square regression fit for parametric study.

*KEY WORDS: spatial variation, response spectrum, dense instrument arrays, statistical study, regression analysis*

## 1. INTRODUCTION

It has been noted in a large number of post earthquake field surveys of damage that for any earthquake event, the degree of damage suffered by similar structures varies appreciably from one location to another, even though the separation between the two structures may be reasonably small. This variation seems to be caused by differences in ground motion. In structural design based on the reliability theory, the variance of earthquake force is as important as the mean value itself. For long span structures and embedded lifeline structures also, the spatial variation of response acceleration has important implications. Therefore we attempt to examine the spatial variation of earthquake damage potential statistically. The acceleration response spectrum has been used as an indicator of the damage potential as it reflects both the effect of amplitude as well as frequency content of the ground motion.

Strong motion records from 39 events for Chiba array in University of Tokyo, Japan and 40 events for SMART-1 array, in Taiwan having a range of magnitudes, epicentral distances and focal depths have been considered. Response analysis is carried out for discrete structural time periods of 0.1, 0.2, 0.5, 1.0 and 2.0 sec. For an earthquake event, the ratio of ordinates of acceleration response spectra for a certain time period, for any station pair, reflects the spatial variation in response over the separation distance.

Even when we consider intra-event variation only, there is appreciable change in response acceleration amplitudes among stations, although the two sites are more or less uniform. To identify the causal factors, regression analysis is used for both the arrays with ratio of response spectra as criterion and station separation, structural time period, damping coefficient, magnitude, epicentral distance, focal depth and earthquake components as variables.

## 2. PROBABILITY DENSITY FUNCTION OF SPECTRAL ORDINATE RATIO

To study the spatial variation of response spectra, the ratios of spectral ordinates at a time period for all possible combinations of stations in an array were calculated for each event and then analysed statistically. The ratios were calculated as (smaller value) / (larger value) for all station pairs rather than taking the response of any one station as the base response. Apparently, the closer the values of the ratio are to 1, the higher is the correlation among the two concerned spectral ordinates.

Since most structures have the damping ratio in the range of 0.05 to 0.20, two values of the damping ratio viz. 0.10 and 0.20 were used in this study in order to examine its effect on response spectra ratio. Scatter plots were then made for each component and for each time period separately using these two damping ratio values.

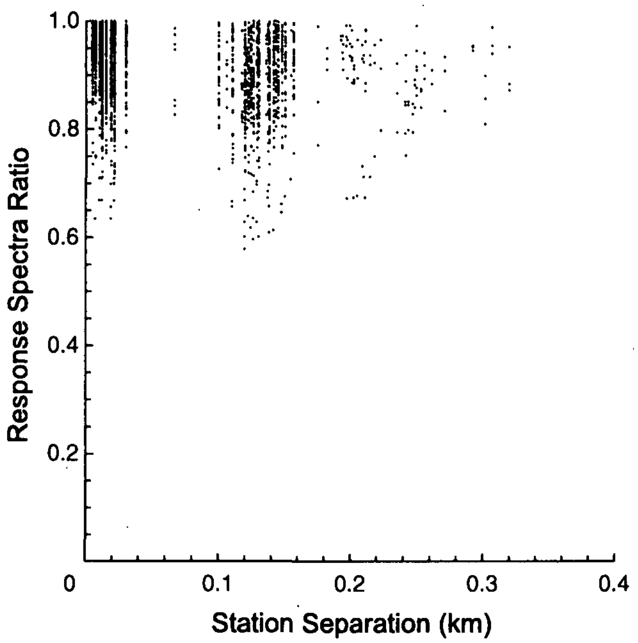

**Fig. 1** Scatter plot of response spectra vs. station separation for Chiba array, EW-component, damping = 0.10, structural time period = 0.50 sec.

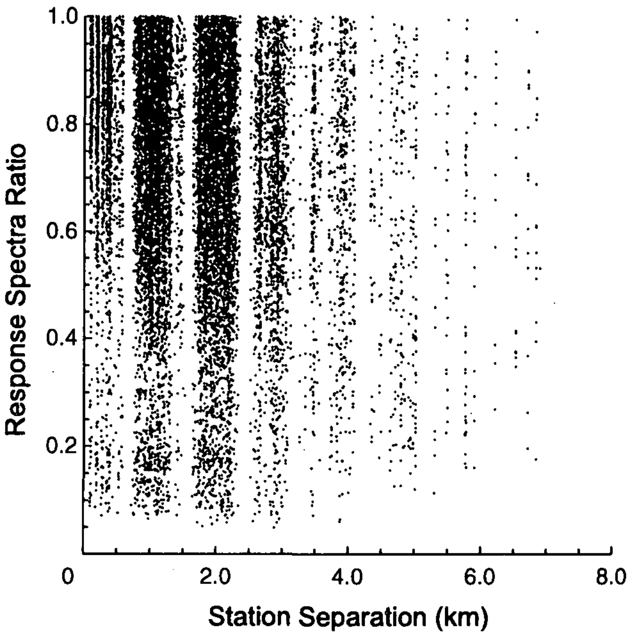

**Fig. 2** Scatter plot of response spectra vs. station separation for SMART-1 array, EW-component, damping = 0.10, structural time period = 0.50 sec.

The typical scatter plots for the EW component with a 0.10 damping ratio as well as 0.5 sec time period are shown in **Fig. 1** for Chiba array and in **Fig. 2** for SMART-1 array. It can be readily seen that there is a very large scatter in the results, especially for SMART-1 array, and that the spectra ratios in Chiba array tend to be higher than those for SMART-1 array. While the values of response spectra ratio in case of Chiba array are generally higher than 0.6 for any case, for SMART-1 array, a reasonable number of spectra ratio values can be observed as low as 0.1 or less.

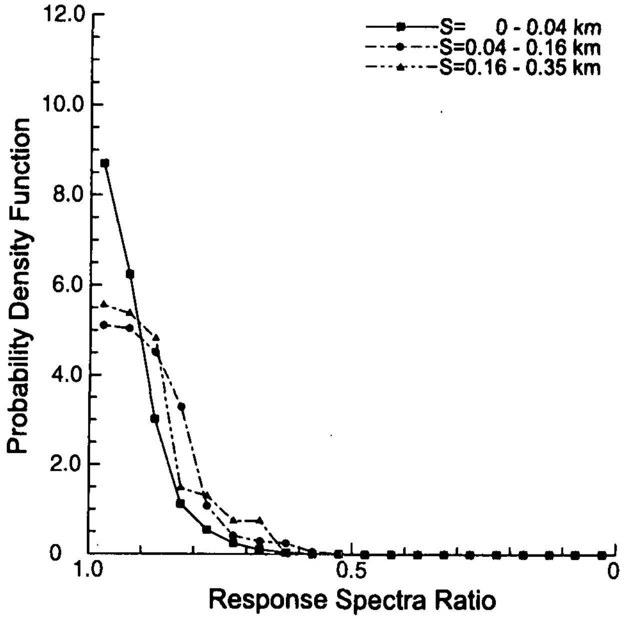

**Fig. 3** Probability density function for different station separation ranges for Chiba array, EW-component, damping = 0.10, structural time period = 0.50 sec.

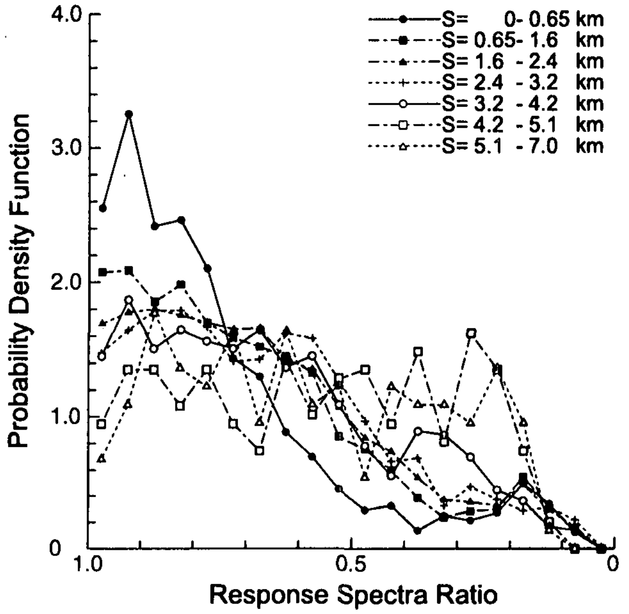

**Fig. 4** Probability density function for different station separation ranges for SMART-1 array, EW-component, damping = 0.10, structural time period = 0.50 sec.

To analyse the distribution of spectral ordinate ratio with station spacing and structural time period, the probability density function is calculated from the scatter plots for various station separation ranges, considering ratio increments of 0.05. These values are then plotted against the mean ratio of each ratio range. For Chiba array, the station separation ranges are 0- 40 m, 40- 160 m, and larger than 160 m. In case of SMART-1 array, the station separation ranges were 0- 650 m, 650- 1600 m, 1600- 2400 m, 2400- 3200 m, 3200- 4200 m, 4200- 5100 m and larger than 5100 m.

The typical probability density function plots for the EW component records of Chiba array and SMART-1 array for structural time period 0.5 sec with 0.10 damping ratio are shown in **Figs. 3** and **4**. Following observations are made:

1) Compared to Chiba array, the probability density function plot for SMART-1 against response spectra ratio is flatter. This indicates a generally lower correlation among response spectra for SMART-1 than for Chiba array. This is attributable mainly to larger station separation for SMART-1 array while site specific effects also have their contribution.

2) There is an inverse relationship between ratio of response spectra and station spacing. The curve for the shortest station separation range (S= 0- 0.650 km) for SMART-1 shows a general shape which is different from curves for larger spacings and similar to Chiba curves. As station separation increases, the peaks of probability distribution become lower and the curves generally flatter, reflecting the reduction in correlation among spectral ordinates.

## 3. STATISTICAL MEAN OF RATIO OF SPECTRAL ORDINATES

From the probability density function curves that are drawn for each structural time period, the statistical mean and the coefficient of variation of the spectral ordinate ratio can be worked out. The plots of the statistical mean of the spectral ordinate ratio against the structural time period are shown in **Fig. 5** for Chiba array and SMART-1 array.

## 4. REGRESSION ANALYSIS

Multiple regression analysis was done separately for the two arrays, Chiba and SMART-1 to observe the influence of the causal factors for the two. To incorporate the effect of qualitative variables like earthquake component and the structural time period, suitable dummy variables have been introduced.

The input data for the regression analysis was generated by calculating ratios of acceleration response spectra for all possible combinations of stations event- wise. Two values of damping ratio, 0.10 and 0.20 were used. The coefficients were then calculated by regression analysis of the data using a least squares method. A linear relationship between the various variables would be ;

$$R=a_0+a_{UD}UD+a_{EW}EW+a_{NS}NS+a_H H+a_M M+a_D D$$
$$+a_L L+a_{T01}T_{01}+a_{T02}T_{02}+a_{T05}T_{05}+a_{T10}T_{10}+a_{T20}T_{20}+a_S S \quad (1)$$

where, $R$ is ratio of response spectra, $a_0$ is constant term in linear regression equation and $a_{UD}$ to $a_S$ are the coefficients of various variables to be calculated through linear regression. $UD$, $EW$ and $NS$ are the dummy variables for the qualitative variable earthquake component. $H$ is damping ratio, $M$ is earthquake magnitude, $D$ is focal depth of earthquake (km) and $L$ is epicentral distance (km). In any data line $T_{01}$, $T_{02}$, $T_{05}$, $T_{10}$ and $T_{20}$ are the dummy variables for the qualitative variable time period, having a value of 1 for the one relevant structural time period out of the five periods 0.1, 0.2 , 0.5, 1.0 and 2.0 sec, and 0 for the other four. $S$ is separation distance for a station pair in km.

However, making both $EW$ and $NS$ equal to 0 simultaneously is a sufficient condition for the vertical component, and also, for the structural time period of 2.0 sec, making the four variables $T_{01}$ - $T_{10}$ equal to 0 is a sufficient condition. Thus, to avoid the perfect multi- collinearity problem, the value of coefficients $a_{UD}$ and $a_{T20}$ are assigned the value of 0 as the base category and other coefficients calculated through regression analysis. The coefficients of the other dummy variable terms would then indicate the values with reference to the base category. The regression model is therefore reduced to;

$$R=a_0+a_{EW}EW+a_{NS}NS+a_H H+a_M M+a_D D+a_L L$$
$$+a_{T01}T_{01}+a_{T02}T_{02}+a_{T05}T_{05}+a_{T10}T_{10}+a_S S \quad (2)$$

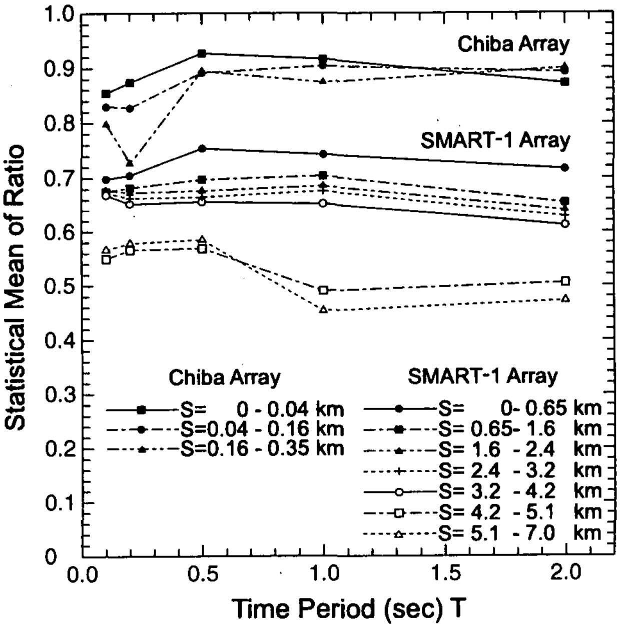

**Fig. 5** Statistical mean of response spectra ratio vs. time period for Chiba array and SMART-1 array, EW-component, damping = 0.10.

The regression analysis was done using full Chiba data, full SMART-1 data and finally a subset of SMART-1 data (station spacings 0.08- 0.35 km). For comparing the effect of various parameters, each of the parameters was normalised by subtracting their mean values and dividing by their standard deviation. The regression equation in its final form is then;

$$R=a'_0+a'_{EW}EW'+a'_{NS}NS'+a'_HH'+a'_MM'+a'_DD'$$
$$+a'_LL'+a'_{T01}T'_{01}+a'_{T02}T'_{02}+a'_{T05}T'_{05}+a'_{T10}T'_{10}+a'_SS' \quad (3)$$

where the coefficients and normalised parameters are, typically;

$a'_{EW}=\sigma_{EW}*a_{EW}$ and $EW'=(EW-\mu_{EW})/\sigma_{EW}$ etc.

The values of constant term and various coefficients in Eq. (3) are shown as a summary plot in **Fig. 6** with the error bars indicating the standard regression error for each parameter.

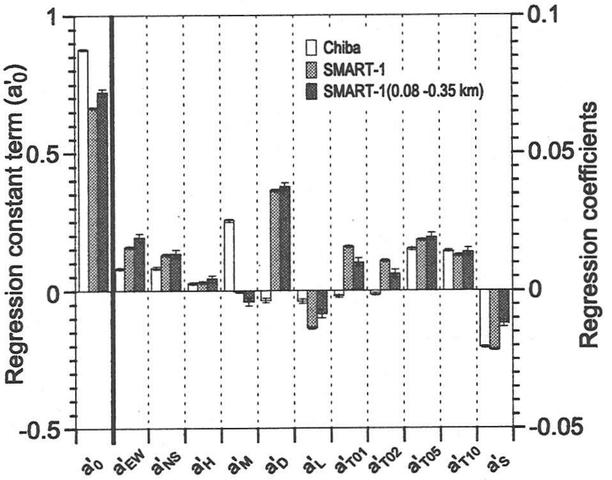

**Fig. 6** Summary plot for regression equation coefficients with error bars showing standard error of regression.

## 5. CONCLUSIONS

Spatial variation of acceleration response spectra is examined using strong motion records for a large number of events from dense accelerometer arrays at Chiba in Japan and SMART-1 in Lotung, Taiwan. The effects of earthquake component, structural damping, earthquake magnitude, focal depth, epicentral distance, structural time period, and station separation on the intra- event variation of response spectra are examined first through an empirical analysis and then through a least square regression fit for parametric study.

A very large scatter of the response spectra ratio is observed for both arrays, especially for SMART-1 array. The mean values of the ratio vary from 10 to 20% for Chiba array while they vary from 25 to 50% for SMART-1 array. The coefficients of variation of the ratio range from 5 to 25% for Chiba array and 30 to 50% for SMART-1 array. The correlation among response spectra is found to be inversely proportional to station separation and shows frequency dependence. For larger time periods, the correlation is lower and not higher. The correlation is also lower for UD earthquake component as compared to the two horizontal components. For higher damping ratio, the correlation among spectra is higher. The effect of the earthquake magnitude, focal depth and epicentral distance on the spatial variation is complex. The three parameters having implicit interdependence, considering their combined effect, a positive contribution to the value of ratio of response spectra is observed in case of larger earthquake events. Furthermore, as mentioned above, the spatial variation for SMART-1 array is much larger than that for Chiba array. This difference can be attributed mainly to the difference in distance between the instruments in the two arrays. However, some of the difference is considered to be due to site specific characteristics.

## ACKNOWLEDGMENTS

The authors wish to thank the Association for Earthquake Disaster Prevention of Japan and National Geophysical Data Center in U.S.A. for publishing the strong motion array record database and to the University of Tokyo for providing the strong motion records.

## REFERENCES

1) Katayama, T., Yamazaki, F., Nagata, S., Lu, L. and Turker, T.: Development of strong motion database for the Chiba seismometer array, Report No. 90-1 (14), Earthquake Disaster Mitigation Engineering, Institute of Industrial Science, University of Tokyo, 1990.

2) Katayama, T., Yamazaki, F., Nagata, S., Lu, L. and Turker, T.: A strong motion database for the Chiba seismometer array and its engineering analysis, *Earthquake eng. struct. dyn.* 19, 1089 – 1106, 1990.

3) Bolt, B. A., Loh, C. H., Penzien, J., Tsai, Y. B. and Yeh, Y. T.: Preliminary report on the SMART 1 strong motion array in Taiwan, Report No. UCB/ EERC- 82/ 13, Earthquake Engineering Research Center, University of California, Berkeley, CA, 1982.

4) Database Advisory Committee and Working Sub-committee: Strong motion array record database- data manual, Association for Earthquake Disaster Prevention of Japan, Vol. A01, July1992.

5) Kawakami, H. and Sharma S.: Statistical study of spatial variation of response spectrum using free field records of dense strong motion arrays, *Earthquake eng. struct. dyn.* 1999.